Elementary Row Operations

Definition: The following three operations on the rows of any matrix are called elementary row operations.

1. Interchange any two rows. $R_i \leftrightarrow R_j$
2. Multiply any row by a nonzero scalar. $R_i := kR_i$
3. Add any scalar multiple of a row to another row. $R_j := R_j + kR_i$

$$\begin{cases} a_1x + a_2y + a_3z = b_1 \\ c_1x + c_2y + c_3z = b_2 \\ d_1x + d_2y + d_3z = b_3 \end{cases} \qquad \left[\begin{array}{ccc|c} a_1 & a_2 & a_3 & b_1 \\ c_1 & c_2 & c_3 & b_2 \\ d_1 & d_2 & d_3 & b_3 \end{array}\right]$$

Example 5: Consider the augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 1 \\ 1 & 2 & 1 & -1 & -2 \\ 2 & 1 & -2 & 0 & 0 \end{array}\right] \tag{1}$$

Give the augmented matrix that is the result of adding -1 times the first row to the second row and -2 times the first row to the third row.

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 1 \\ 1 & 2 & 1 & -1 & -2 \\ 2 & 1 & -2 & 0 & 0 \end{array}\right] R_2 := R_2 - R_1 \left[\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 0 & -3 \\ 2 & 1 & -2 & 0 & 0 \end{array}\right] R_3 := R_3 - 2R_1 \left[\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 0 & -3 \\ 0 & -1 & -6 & 2 & -2 \end{array}\right]$$

Definition: We call two matrices with the same number of rows and columns row equivalent if there is a sequence of elementary row operations that converts one matrix into the other.

Example 6: Are the matrices $\overset{A}{\begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}}$ and $\overset{B}{\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}}$ row equivalent? Explain.

Yes. $A \xrightarrow{R_1 \leftrightarrow R_3} B$

$\frac{1}{2}R_2 =: R_2$